PROBLEMS.

52. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

In how many ways can we arrange 12 friends of the MONTHLY, around a table, so that; (1), the editors may never be together, (2), Matz and Halsted may never be apart; and (3), Zerr and Ellwood may always have Gruber betwixt them?

53. Proposed by LEONARD E. DICKSON, M.A., Fellow in Mathematics, University of Chicago.

Can it be proven that the value of the expression

$$\Big\{5x-\frac{5x+3}{3}\cdot\frac{5x(5x-1)}{1.2}+\frac{(5x+3)(5x+8)}{4.5}\cdot\frac{5x(5x-1)(5x-2)}{1.2.3}$$

$$-\frac{(5x+3)(5x+8)(5x+13)}{5.6.7}\cdot$$

$$\frac{5x(5x-1)(5x-2)(5x-3)}{1.2.3.4}+\frac{(5x+3)\ldots(5x+18)}{6.7.8.9}\cdot\frac{5x\ldots\ldots(5x-4)}{1\ldots5}+\ldots$$

$$+(-1)^{5x-2}\frac{(5x+3)\ldots(30x-12)}{5x\cdot(5x+1)\ldots(10x-3)}\cdot5x+(-1)^{5x-1}\frac{(5x+3)\ldots(30x-7)}{(5x+1)\ldots(10x-1)}\Big\}$$

is identically zero?

GEOMETRY.

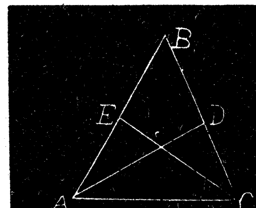
Conducted by B.F.FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEM.

42. Proposed by G. I. HOPKINS, Instructor in Mathematics and Physics in High School, Manchester, New Hampshire.

If the bisectors of two angles of a triangle are equal the triangle is isosceles.

II. Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia; H. C. Whitaker, M. S., M. E., Professor of Mathematics. Manual Training School, Philadelphia, Pennsylvania; WILLIAM HOBBY, a Student in University of Tennessee, Knoxville, Tennessee.



Let $AC=b$. Then $AD=b\sin C/\sin(C+\frac{1}{2}A)$,

$CE=b\sin A/\sin(A+\frac{1}{2}C)$.

$\therefore\sin C\sin(A+\frac{1}{2}C)=\sin A\sin(C+\frac{1}{2}A)$.

Let $x=\sin\frac{1}{2}A$, $y=\sin\frac{1}{2}C$.

Then $(2xy-2x^3y-y^2+2x^2y^2)\sqrt{1-y^2}$

$$=(2xy-2xy^3-x^2+2x^2y^2)\sqrt{1-x^2}.$$

$\therefore(x^2-y^2)\{x^2+y^2-x^4-x^2y^2-y^4-4xy$
$+4x^3y+4xy^3-4x^3y^3\}=0.$

$\therefore x=y$ and $\angle A=\angle C$,

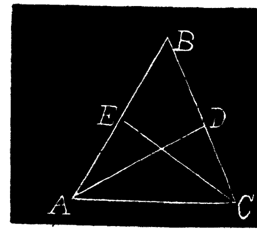
also, $(x-y)^4-(x-y)^2+xy(1-xy)+xy(1-2xy)^2=0$.

III. Solution by J. H. GROVE, Howard Payne College, Brownwood, Texas.
(Solution by Reductio ad Absurdum.)

Suppose the $\triangle$ to be scalene and that $AB>BC$.

(1) $AB.AC=BD.DC+AD^2$

(2) $BC.AC=BE.AE+CE^2$



Then (1) $AB.AC-BD.DC=AD^2$ and

(2) $BC.AC-BE.AE=CE^2$. But $AD^2=CE^2$. Hyp.

$\therefore AB.AC-BD.DC=BC.AC-BE.AE$.

But $AB.AC>BC.AC$ (Hyp.), and

$BD.DC<BE.AE$ ($\angle a<\angle c$ Hyp.)

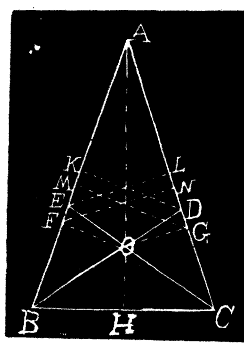
($BD<BE$ and $DC<AE$)

$\therefore$ The conclusion above reached: $AB.AC-BD.DC=BC.AC-BE.AE$ is absurd. It can be true only in case the $\triangle$ is isosceles.

$\therefore$ If the bisectors AD and CE are equal, $AB=BC$. *Q. E. D.*

IV. Solution by EDW. R. ROBBINS, Master in Mathematics of Lawrenceville School, Lawrrnceville, New Jersey.

Let ABC be a $\triangle$ of which EC and BD are equal bisectors of base angles. To prove the $\triangle$ is isosceles. Draw third bisector AO. Draw perpendiculars OH, OF, OG: these are equal lines. From E, D, F, G, draw perpendiculars to opposite side.



In $\triangle$s AFO, AOG: $FO=OG$: $AO=AO$ and angles at A are equal.

Therefore these triangles are equal right triangles, and $AF=AG$. In right $\triangle$s AGM and AFN angle at A is common and $AF=AG$. Hence $\triangle$s are equal and $FN=GM$.

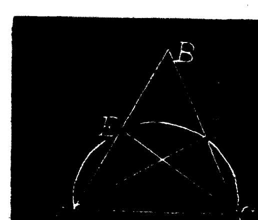
Now the right triangles AEL and ADK are similar respectively to AFN and AGM. But these last $\triangle$'s are equal and hence the $\triangle$'s AEL and ADK are equal. And therefore $AE=AD$.

The triangles ADB and AEC have two sides of the one, AD, DB equal respectively to two sides of the other AE, EC, also the angle at A is common. Hence the $\triangle$s are equal. Therefore $AB=AC$ and original $\triangle ABC$ is isosceles.

Q. E. D.

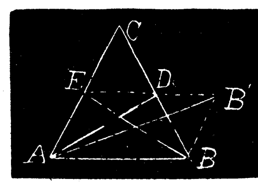
V. Solution by P. S. BERG, Apple Creek, Ohio.

Let ABC be a triangle, $AD=CE$. Through the three points A, C and E pass the circumference of a circle. It will also pass through D. For if it meets AD ni P between A and D the arc EA must be greater than PE, since ECA which is equal to DCE is greater than PCE. Also the arc PE is equal to the arc PC, since the angle EAP is equal to the angle PAC. Whence the arc AEP is greater than



EPC, and consequently the chord AP is greater than the chord CE. But by hypothesis $CE=AD$; then AP is greater than AD which is impossible. Hence the circle which passes through A, C, and E cannot cut AD between A and D. In like manner it can be shown that the circle cannot cut AD beyond D. Hence it must pass through D. Hence the angle EAD which is half of A is equal to DCE which is half of C. Therefore the angle A is equal to the angle B and the triangle is isosceles. *Q. E. D.*

VI. Solution by W. W. MOSS, Instructor in Mathematics, Brown University, Providence, Rhode Island.



Let AD and BE be equal bisectors of the $\angle$s CAB and CBA of the $\triangle ABC$. To prove $\triangle ABC$ isosceles. Move the $\triangle DBA$ so that the side AD will coincide with its equal EB, A falling at E, D at B. Then fold $\triangle BAD$ upon AD (EB) as axis till B falls upon the plane at B', $\triangle BAD$ taking position EBB'. Draw AB'. Consider the $\triangle$s AEB' and ABB'; $\angle AEB' = \angle AEO + \angle OEB' = \angle AEO + \angle OAB = \angle AEO + \angle OAE = \angle AOB$. $\angle ABB' = \angle ABE + \angle EBB' = \angle OBD + \angle ODB = \angle AOB$. $\therefore$ $\angle AEB' = \angle ABB'$. $\angle CAB + \angle CBA < 180^\circ$ and halving $\angle OAB + \angle OBA < 90^\circ$. $\therefore$ $\angle AOB = 180^\circ - (\text{angle } OAB + \text{angle } OBA) > 90^\circ$. $\therefore$ angles AEB' and ABB' are obtuse and equal side EB'=side AB and $AB'=AB'$. $\therefore$ $\triangle AEB' = \triangle ABB'$ having two sides and an opposite angle in one equal to homol. parts in the other, the equal angles being obtuse. $\therefore$ angle EAB=angle EAB'+angle BAB'=angle $AB'B$ +angle $AB'E$=angle $EB'B$=angle DBA.

$\therefore$ $\triangle ABC$ is isosceles. *Q. E. D.*

Solutions of this problem were received from *J. C. CORBIN, WILLIAM PARKISON, F. P. MATZ. O. W. ANTHONY, A. M. HUGHLETT, H. W. DRAUGHON and J. F. W. SCHEFFER. Professor SCHEFFER sent in three solutions and Professor GROVE two.*

Note.—An excellent demonstration of this proposition is given on page 44, of *Dr. Halsted's Elementary Synthetic Geometry.*

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

32. Proposed by J. F. W. SCHEFFER, Hagerstown, Maryland.

Suppose it to be possible to perform the passage through the north pole: at